

# DESIGN OF ROUND-TRIP TRAJECTORIES TO NEAR-EARTH ASTEROIDS UTILIZING A LUNAR FLYBY

Sonia Hernandez\* and Brent W. Barbee†

There are currently over 7,700 known Near-Earth Asteroids (NEAs), and more are being discovered on a continual basis. Current models predict that the actual order of magnitude of the NEA population may range from  $10^5$  to  $10^6$ . The close proximity of NEA orbits to Earth's orbit makes it possible to design short duration round-trip trajectories to NEAs under the proper conditions. In previous work, 59 potentially accessible NEAs were identified for missions that depart Earth between the years 2016 and 2050 and have round-trip flight times of a year or less. We now present a new method for designing round-trip trajectories to NEAs in which the Moon's gravity aids the outbound trajectory via a lunar flyby. In some cases this gravity assist can reduce the overall spacecraft propellant required for the mission, which in turn can allow NEAs to be reached which would otherwise be inaccessible to a given mission architecture. Results are presented for a specific case study on NEA 2003 LN<sub>6</sub>.

## INTRODUCTION

There are currently over 7,700 known Near Earth Asteroids (NEAs) and more are being discovered on a continual basis\*. NEAs are small (often rocky but occasionally metallic) celestial bodies with mean diameters ranging from several meters to several kilometers. Their solar orbits are such that they often closely approach Earth's orbit, and this makes them a potential threat to life on Earth by virtue of collision hazards.<sup>1</sup> However, this also presents a unique opportunity for exploration. Because NEAs have orbits in close proximity to Earth's orbit, it is possible to design short duration round-trip trajectories to NEAs, which may permit these objects to become destinations for human missions.<sup>2,3</sup>

There are a number of compelling reasons to send missions to NEAs. Asteroids have remained largely unchanged in composition and studying them could provide insight into the origins of our solar system. Moreover, because an NEA could be on a collision course with Earth in the near future, understanding their characteristics is vital. Additionally, a human mission to an NEA would be the most ambitious journey ever undertaken by our space program and would renew passion for space exploration among both students and the general public. Indeed, the current presidential administration has proposed, as part of the Flexible Path program, that such a mission be launched during the mid-2020s†.

---

\*Graduate Student, Department of Aerospace Engineering and Engineering Mechanics, The University of Texas at Austin, Austin, TX, 78712, USA, [sonia.hernandez@mail.utexas.edu](mailto:sonia.hernandez@mail.utexas.edu).

†Flight Dynamics Engineer, NASA Goddard Space Flight Center, Greenbelt, MD, 20771, USA, [brent.w.barbee@nasa.gov](mailto:brent.w.barbee@nasa.gov).

\*<http://neo.jpl.nasa.gov/stats/>

†<http://www.space.com/news/obama-space-plan-speech-100415.html>

In previous work, a study was performed to identify NEAs that may be accessible for round-trip missions that employ a single heavy-lift launch, depart Earth between the years 2016 and 2050, and have a total round-trip flight time of less than a year.<sup>4</sup> The algorithm developed for that study identified 59 accessible NEAs and 10 marginally inaccessible NEAs. This parametric study was performed with two-body dynamics and patched conics between the Earth and the Sun for the spacecraft trajectory segments, though full high-precision ephemerides were used for the Earth and the NEAs.

In this paper we present a newly developed method for designing round-trip trajectories to NEAs in which the Moon's gravity aids the outbound segment of the trajectory via a lunar flyby. The goal is to perform the flyby in a manner that reduces the overall required propellant mass for the mission, which can cause previously inaccessible NEAs to become accessible (assuming no other changes to the mission architecture). We demonstrate the efficacy of this methodology by extending the work in Reference 4 to include the Moon's gravity in a three-body model. This is combined with an automated algorithm that performs a parametric scan to target round-trip trajectories to NEAs utilizing a lunar flyby in a manner that serves to reduce the required propellant mass for the mission.

This paper is organized as follows. The first section introduces a three-body model, centered at Earth and perturbed by the Moon, which serves as a first estimate to design lunar flybys. In the next section, we describe the automated algorithm used to design round-trip trajectories to NEAs. This algorithm is implemented by combining the model described in the first section with patched conics between the Earth and the Sun. The last section deals with implementing Moon's accurate ephemerides into the algorithm and an example trajectory mission is shown for NEA 2003 LN<sub>6</sub>.

### **Disclaimer Regarding NEA Accessibility**

The discussion we present in this paper often hinges on whether a NEA is classified as "accessible" or "inaccessible" or "marginally inaccessible" by our algorithms. It is important to note that the meaning of "accessible" in the context of human missions to NEAs is currently an area of vigorous research and we are making no claims whatsoever regarding whether any NEAs are truly accessible for human exploration. Rather, we are attempting to identify *potentially* accessible NEAs from a dynamics point of view, and this what we mean when talk about "accessibility." However, while the trajectory design results may make a particular NEA appear accessible for a given mission architecture (mass of crew vehicle, number of launches, type of propulsion system, etc.), there are a host of other constraints and considerations (e.g., various operational constraints and human health and safety constraints) that must be analyzed in great detail before the accessibility of a NEA can be truly and properly evaluated. Those other aspects of human space flight to NEAs are quite complex and are thus well outside the scope of this paper. So, to prevent our verbiage from becoming unwieldy we will simply talk about NEA "accessibility" in the context of our current dynamical analysis without repeating these caveats, under the assumption that the reader has read this disclaimer and understands what we mean by "accessible."

### **LUNAR FLYBY IN A THREE-BODY MODEL**

The purpose of this research is to design trajectories to NEAs which utilize a lunar flyby in the outbound (Earth departure) segment of the trajectory. The first step in designing the trajectory consists of targeting a lunar flyby that will cause the trajectory to intercept the NEA. To accomplish this, we introduce a dynamics model for the spacecraft which includes the effects of lunar gravity in a non-rotating Earth-centered frame.

The acceleration experienced by the spacecraft, with Earth as the primary central body and the Moon as a perturbing body is<sup>5</sup>

$$\ddot{\mathbf{r}} = -\frac{\mu_e}{\|\mathbf{r}\|^3}\mathbf{r} - \mu_m \left( \frac{\mathbf{r} - \mathbf{r}_m^e(t)}{\|\mathbf{r} - \mathbf{r}_m^e(t)\|^3} + \frac{\mathbf{r}_m^e(t)}{\|\mathbf{r}_m^e(t)\|^3} \right), \quad (1)$$

where  $\mathbf{r} \in \mathbb{R}^3$  is the position of the spacecraft with respect to the Earth and  $\mathbf{r}_m^e(t) \in \mathbb{R}^3$  is the position of the Moon with respect to the Earth at any given epoch. The gravitational parameters of the Earth and the Moon are,  $\mu_e$  and  $\mu_m$ , respectively\*. A numerical integration algorithm is used to integrate Eq. (1) by defining a state vector

$$\mathbf{x} = \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix} \in \mathbb{R}^6$$

for the spacecraft with initial conditions  $\mathbf{x}_0 = [\mathbf{r}_0 \ \mathbf{v}_0]^T$ , where  $\mathbf{v}_0 \in \mathbb{R}^3$  is the velocity at time  $t_0$ .

We obtain an initial trajectory estimate by modeling the motion of the Moon about the Earth with a simplified planar, circular model in which the mean motion of the Moon is

$$n_m = \sqrt{\frac{\mu_e}{a_m^3}}$$

with  $a_m = 384,400$  km (the mean distance between the Earth and the Moon). In this model, the position of the Moon,  $\mathbf{r}_m^e(t)$ , at any epoch  $t$  is

$$\mathbf{r}_m^e(t) = a_m \begin{bmatrix} \cos(\alpha_t) \\ \sin(\alpha_t) \\ 0 \end{bmatrix}. \quad (2)$$

The angle  $\alpha_t$  is measured from the positive x-axis of the Earth-Moon line and  $\alpha_{t_0}$  represents the initial location of the Moon when the spacecraft departs Earth (see Figure 1).  $\alpha_t$  at any epoch is

$$\alpha_t = \alpha_{t_0} + n_m t \quad (3)$$

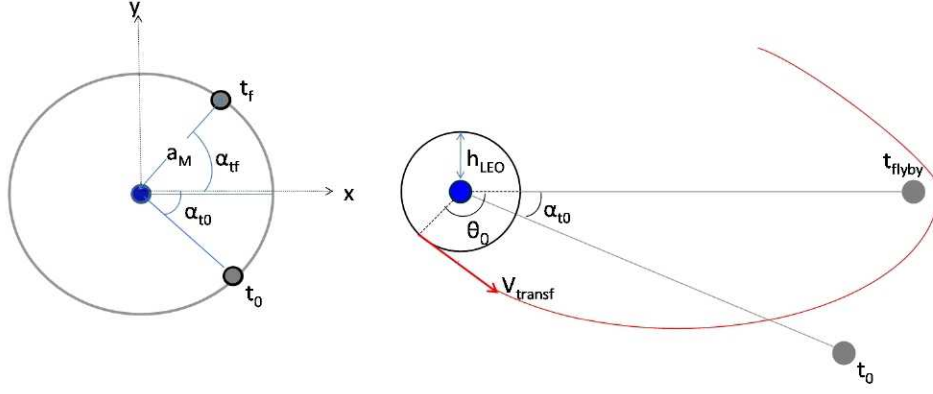
For the purposes of this study, we make the assumption that the spacecraft departs from a circular Low Earth Orbit (LEO) at an altitude,  $h_{LEO}$ , of 185 km, and travels to the Moon on a Hohmann transfer. However, the design of the algorithm is such that any  $h_{LEO}$  or any type of transfer may be used. The initial impulse required to arrive at the Moon on this trajectory is

$$\begin{aligned} \Delta V_{LEO} &= v_{trans} - v_0 \\ &= \sqrt{2\mu_e \left( \frac{1}{r_{LEO}} - \frac{1}{2a_m} \right)} - \sqrt{\frac{\mu_e}{r_{LEO}}} \\ &= 3.1352 \text{ km/s} \end{aligned} \quad (4)$$

We also need to determine where the Moon should be initially located so that the spacecraft will fly past the Moon in a manner that provides additional velocity change sufficient to cause the

---

\*The values for the gravitational parameters are  $\mu_e = 3.986 \cdot 10^5 \text{ km}^3 \text{ s}^{-2}$  and  $\mu_m = 4.903 \cdot 10^3 \text{ km}^3 \text{ s}^{-2}$ .



**Figure 1. Schematic of Lunar Flyby in a Model Centered at the Earth and Perturbed by the Moon.**

spacecraft to reach Earth's Sphere of Influence ( $SOI_e$ ). After imparting on the spacecraft located in LEO the initial impulse  $\Delta V_{LEO}$ , its position and velocity are

$$\mathbf{r}_0 = \begin{bmatrix} r_{LEO} \cos(\theta_0) \\ r_{LEO} \sin(\theta_0) \end{bmatrix} \text{ and } \mathbf{v}_0 = \begin{bmatrix} -(v_0 + \Delta V_{LEO}) \sin(\theta_0) \\ (v_0 + \Delta V_{LEO}) \cos(\theta_0) \end{bmatrix}, \quad (5)$$

where the angle  $\theta_0$  defines the initial location of the spacecraft from the positive x-axis of the Earth-centered inertial frame at time  $t_0$  and  $r_{LEO} = r_e + h_{LEO}$  (with  $r_e = 6,378.137$  km).

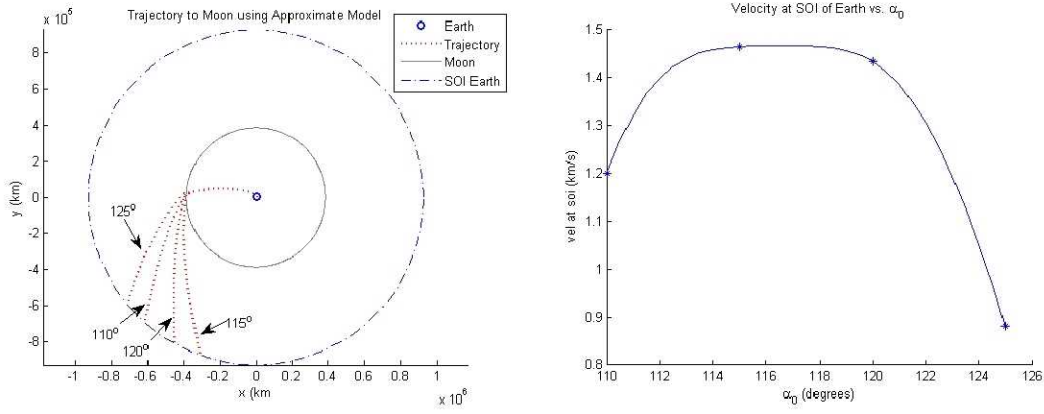
An algorithm was created using the model described in Eq. (1) to search for all the possible angles  $\alpha_{t_0}$  such that the lunar gravity assist would cause the spacecraft to escape Earth and reach  $SOI_e$ . The result of this search (for trajectories with Earth departure characteristics as defined in Eq. (4)) is that the location of the Moon (at the time at which the spacecraft departs Earth) must be within the interval

$$\theta_0 + 110^\circ \leq \alpha_{t_0} \leq \theta_0 + 125^\circ \quad (6)$$

This means that for initial angles  $\alpha_{t_0}$  which are smaller than  $110^\circ$  or greater than  $125^\circ$ , the Moon is too far from the spacecraft when it reaches the distance of  $a_m$  to provide the spacecraft with the necessary gravity assist to escape Earth. As can be observed from Figure 2, the spacecraft will reach  $SOI_e$  with different flight paths and velocities depending on the particular value of  $\alpha_{t_0}$  within the interval given in Eq. (6). Thus, there are families of post-flyby trajectories that may or may not reach the NEA even though they do reach  $SOI_e$ ; merely reaching  $SOI_e$  is a necessary but insufficient condition for subsequent interception of the NEA.

Note that in the approximated planar, circular model described in this section, the angle of departure  $\theta_0$  can vary from  $0^\circ \leq \theta_0 < 360^\circ$  and  $\alpha_{t_0}$  is varied as given by Eq. (6). When searching for trajectories to intercept NEAs, it is necessary first to scan through all the possible combined initial conditions of  $\theta_0$  and  $\alpha_{t_0}$  that will produce a trajectory which reaches  $SOI_e$ . This requires repeatedly numerically integrating Eq. (1), which can be computationally expensive. However, since in this system the Moon rotates at a constant rate about the Earth, it is possible to only integrate for  $\theta_0 = 0$  and any  $\alpha_{t_0}$  as in Eq. (6), which then allows a simple rotation through angle  $\theta_0$  to transform the trajectory to the desired location. If the state obtained with  $\theta_0 = 0$  is  $\mathbf{x}^0 = [\mathbf{r}^0 \ \mathbf{v}^0]^T$ , the





**Figure 2. (a) Possible Flyby Trajectories that Reach  $SOI_e$  (b) Velocity Magnitude of the Spacecraft Upon Reaching  $SOI_e$ .**

associated state at any other  $\theta_0$  will be

$$\mathbf{x}_{\theta_0} = \begin{bmatrix} \mathbf{r}^{\theta_0} \\ \mathbf{v}^{\theta_0} \end{bmatrix} = \begin{bmatrix} \mathbf{R}^T \mathbf{r}^0 \\ \mathbf{R}^T \mathbf{v}^0 \end{bmatrix}, \quad (7)$$

where the transformation matrix  $\mathbf{R}$  is

$$\mathbf{R} = \begin{bmatrix} \cos(\theta_0) & \sin(\theta_0) & 0 \\ -\sin(\theta_0) & \cos(\theta_0) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

## PATCHED CONICS METHODOLOGY

In Reference 4, a study was performed to identify NEAs that may be accessible for round-trip missions which employ a single heavy-lift launch, depart Earth between the years 2016 and 2050, and have a total round-trip flight time of a year or less. The algorithm developed for that study identified 59 accessible NEAs and 10 marginally inaccessible NEAs. The parametric analysis performed in that study utilized two-body dynamics and patched conics between the Earth and the Sun for the spacecraft trajectory segments, though full high-precision ephemerides were used for the Earth and the NEAs.

In this section we extend the work in Reference 4 by designing round-trip trajectories to NEAs in which a lunar gravity assist aids the outbound segment of the trajectory. The goal is to perform the lunar flyby in a manner that reduces the overall required propellant mass for the mission, which can cause NEAs found to be inaccessible in the previous study to become accessible (under the mission architecture assumptions of the previous study).

The profile of a human mission to an NEA utilizing a lunar flyby is shown in Figure 3. The round-trip trajectory consists of four stages: Earth departure to lunar flyby, post-flyby trajectory to NEA arrival, stay time on the NEA's orbit, and NEA departure to Earth return. For the purposes of this study, the Earth departure date, flight times for each stage, and  $\Delta V$  magnitudes were subject to the same constraints specified in Reference 4. Each of the four stages are now detailed in turn.

Stage 1 Shown in Figure 3(a), this stage of the trajectory is designed in an Earth-centered, inertial frame such that the trajectory departs Earth to perform a flyby at the Moon.

Specifying the date of departure  $t_{dep_e}$ , the initial departure angle  $\theta_0$ , and the location of the Moon at departure  $\alpha_{t_0}$  (see Figure 1) fully defines the initial conditions given in Eq. (5), which allows for the integration of Eq. (1). The trajectory is propagated until it reaches  $SOI_e$ , over a time span of  $tof_{soi}$ .

The  $\Delta V_{LEO}$  in Eq. (4) corresponds to a characteristic energy for Earth departure,  $C_3 = -2.0390 \text{ km}^2/\text{s}^2$ . This value is defined as the square of the hyperbolic excess velocity,  $v_\infty$ , with respect to Earth on a hypothetical outbound hyperbola, and is computed from the energy equation of an orbit as<sup>6</sup>

$$E = -\frac{\mu_e}{2a_m} = \frac{v_\infty^2}{2} - \frac{\mu_e}{r_\infty} = \frac{C_3}{2} \Rightarrow C_3 = -2.0390 \text{ km}^2/\text{s}^2. \quad (8)$$

Stage 2 When the trajectory reaches  $SOI_e$  after the flyby, the Earth-centered state  $\mathbf{x}_{soi}^e = [\mathbf{r}_{soi}^e \ \mathbf{v}_{soi}^e]$  is transformed into a Sun-centered frame as shown in Figure 3(b).

$$\begin{aligned} \mathbf{r}_{soi}^s &= \mathbf{r}_{soi}^e + \mathbf{r}_{et_{soi}}^s \\ \mathbf{v}_{soi}^s &= \mathbf{v}_{soi}^e + \mathbf{v}_e^s \end{aligned} \quad (9)$$

where  $\mathbf{x}_e^s = [\mathbf{r}_e^s \ \mathbf{v}_e^s]^T$  is the state of the Earth with respect to the Sun at the time when the trajectory reaches  $SOI_e$ , that is,  $t_{soi} = t_{dep_e} + tof_{soi}$ , which are computed from precise ephemerides files. To simplify the notation, we drop the superscript  $s$  from the remaining states since they are all in a Sun-centered frame. Note that for this trajectory we use a simplified planar, circular Earth-Moon model, and therefore  $\mathbf{x}_{soi}^e \in \mathbb{R}^2$ , however,  $\mathbf{x}_e^s \in \mathbb{R}^3$ . We have also assumed that the Moon's trajectory lies in the ecliptic plane. These simplifying assumptions are absent in the full version of the algorithm, which is detailed in a subsequent section. The true ephemeris of the Moon is utilized in the full version of the algorithm.

A trajectory from the  $SOI_e$  crossing to the NEA is now designed via Lambert targeting by choosing a time to reach the asteroid from  $SOI_e$ ,  $tof_{arra}$ . The trajectory is designed in such a way that the post-flyby maneuver,  $\Delta V_{soi}$  is minimized. Ideally,  $\Delta V_{soi} = 0$ , a constraint that will be added in future work when we perform optimization of the trajectory. This constraint may not always be achievable, but driving  $\Delta V_{soi}$  as close to zero as possible is clearly beneficial.

$$\Delta V_{soi} = \|\mathbf{v}_{dep_{soi}} - \mathbf{v}_{soi}\|, \quad (10)$$

where  $\mathbf{v}_{dep_{soi}}$  is the required velocity at  $SOI_e$  to intercept the NEA and  $\mathbf{v}_{soi}$  is the velocity that the spacecraft has after the lunar flyby when reaching  $SOI_e$ .

At the end of this stage, the spacecraft performs a maneuver,  $\Delta V_{arra}$ , to match the orbit of the NEA upon arrival.

$$\Delta V_{arra} = \|\mathbf{v}_{a_{arr}} - \mathbf{v}_{arra}\|, \quad (11)$$

where  $\mathbf{v}_{a_{arr}}$  is the heliocentric velocity vector of the NEA at the epoch of arrival, which is computed from the JPL HORIZONS ephemeris file and  $\mathbf{v}_{arra}$  is the velocity vector of the spacecraft upon reaching the NEA.

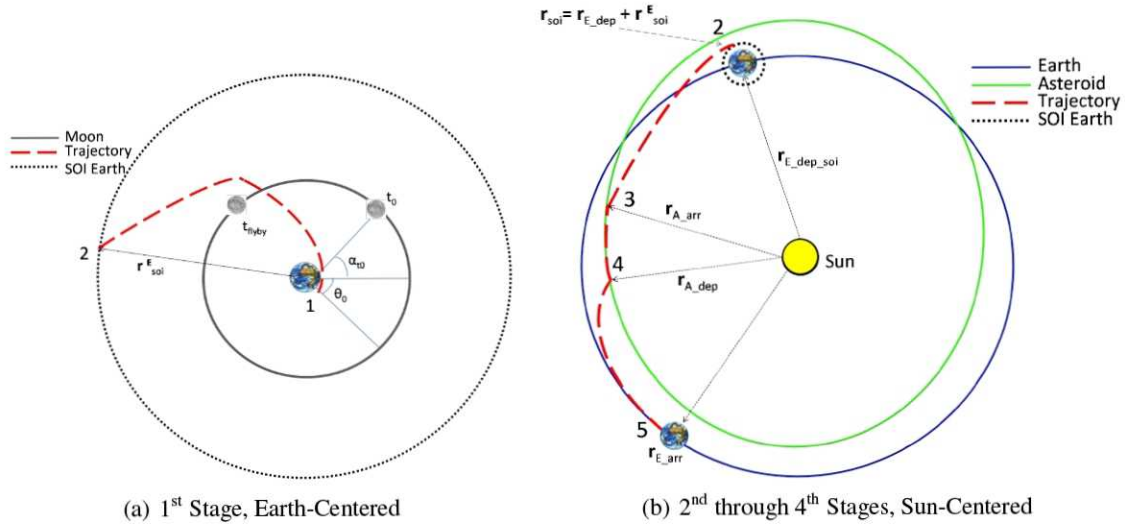
Stage 3 The spacecraft remains with the NEA on its orbit for a selected time span,  $tof_{stay}$ .

Stage 4 After the stay time at the NEA has elapsed, a trajectory is designed to depart the NEA with  $\Delta V_{dep_a}$ , and return the spacecraft to Earth with the desired flight time,  $tof_{arr_e}$ .

$$\Delta V_{dep_a} = \|\mathbf{v}_{dep_a} - \mathbf{v}_{a_{dep}}\|, \quad (12)$$

where  $\mathbf{v}_{a_{dep}}$  is the heliocentric velocity vector of the NEA at the epoch of departure from it, which is again computed from the JPL HORIZONS ephemeris file and  $\mathbf{v}_{dep_a}$  is the velocity vector of the spacecraft before departure from it.

The algorithm also allows for a possible re-entry impulsive maneuver,  $\Delta V_{arre}$  if the spacecraft exceeds the maximum value  $v_{arre_{max}} = 12 \text{ km/s}$ .

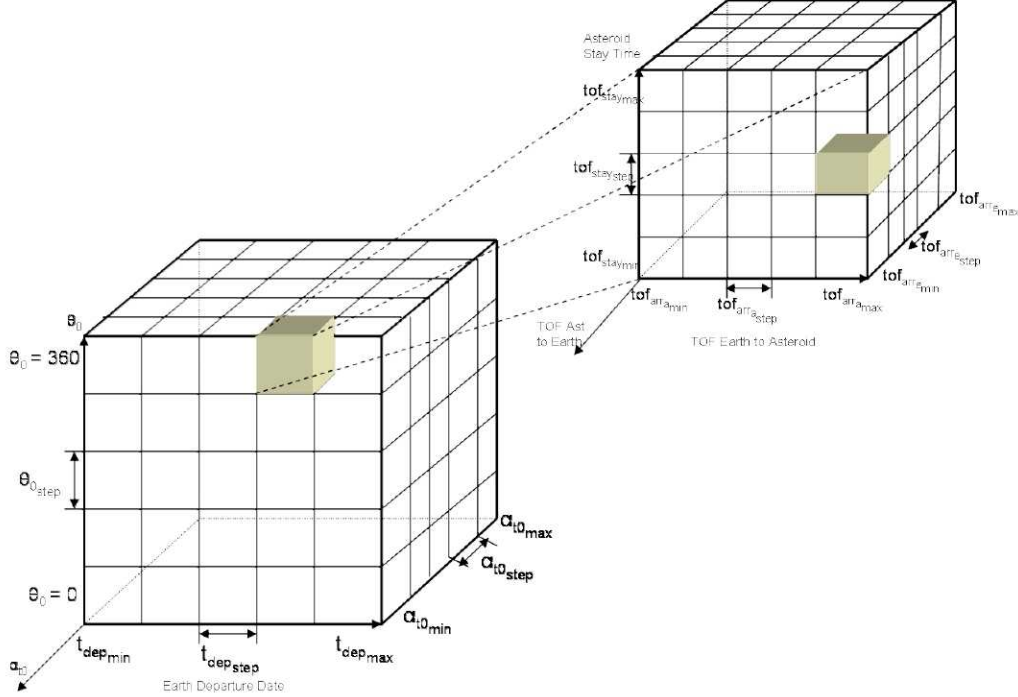


### Figure 3. Trajectory Schematics

We designed the algorithm to use an embedded trajectory grid methodology to assess NEA accessibility by computing all the possible round-trip trajectories. A schematic of the parameter space structure of the algorithm is depicted in Figure 4. The algorithm varies six independent parameters within set bounds in order to identify solutions that satisfy problem constraints. For this paper, we chose to use the same constraints as in Reference 4 so that results could be compared. Whether an NEA is classified as accessible (from a dynamics point of view) depends on maximum allowable round-trip flight time, lower and upper bounds for the trajectory segment flight times, step sizes at which trajectory segment flight times are sampled, lower and upper bounds for the Earth departure date, the step size at which Earth departure date is sampled, and constraints on total spacecraft mass as a function of the assumed launch and propulsion system capabilities. The independent parameters are:

- $t_{dep_e}$ : Date of Earth Departure
- $\theta_0$ : The initial angle at LEO departure
- $\alpha_{t_0}$ : The location of the Moon at departure time, in accordance with Eq. (6)

- $tof_{arr_a}$ : Time of flight from  $SOI_e$  to Asteroid
- $tof_{stay}$ : Time to stay at NEA
- $tof_{arr_e}$ : Time of flight from Asteroid to Earth



**Figure 4. Schematic of Embedded Grid Algorithm**

To evaluate whether the total mission duration constraint is met, the total mission duration is computed as the sum of the flight times associated with each of the four mission stages, according to

$$tof_{tot} = tof_{soi} + tof_{arr_a} + tof_{stay} + tof_{arr_e} \leq 360 \text{ days.} \quad (13)$$

Note that  $tof_{soi}$ , the time of flight to get from  $LEO$  to  $SOI_e$  depends on the specific values chosen for  $\theta_0$  and  $\alpha_{t_0}$ .

Evaluation of mission mass constraints requires the computation of the total mission  $\Delta V$ , which is simply the sum of all the maneuver magnitudes,

$$\Delta V_{tot} = \Delta V_{soi} + \Delta V_{arr_a} + \Delta V_{dep_a} + \Delta V_{arr_e}. \quad (14)$$

In this study, we utilized the launch mass versus  $C_3$  values of the notional Ares V heavy-lift launch vehicle. We assumed that the capabilities of Ares V is representative of a future heavy-lift human-rated launch vehicle that will be available by the mid-2020s.<sup>7</sup> For a  $C_3 = -2.0390 \text{ km}^2/\text{s}^2$ , this corresponds to an available launch mass  $m_{C_3} = 5.5774 \cdot 10^4 \text{ kg}$ . The mass constraint is satisfied if

$$\alpha_m = \frac{m_{req}}{m_{C_3}} \leq 1 \quad (15)$$



where

$$m_{req} = m_{dry} e^{\left(\frac{\Delta V_{tot}}{g I_{sp}}\right)},$$

and  $\Delta V_{tot}$  is in (14). The parameters  $m_{dry} = 17,078 \text{ kg}$  and  $I_{sp} = 314 \text{ s}$  are obtained from the notional parameters of the Orion crew vehicle, which was chosen because it is the most recent design capable of carrying crew on a mission beyond LEO.<sup>4</sup>

### Preliminary Results: NEA 2000 SG<sub>344</sub>

NEA 2000 SG<sub>344</sub> has an absolute magnitude,  $H$ , of 24.8, which yields an estimated physical size of 29 to 66 m for an albedo range of 0.25 to 0.05, respectively. While those albedos are typical for NEAs, it is quite possible for an NEA to have a significantly lower or higher albedo and thus the estimated size based on absolute magnitude and assumed albedo should be treated as very approximate. 2000 SG<sub>344</sub> has semi-major axis  $a = 0.977 \text{ AU}$ , eccentricity  $e = 0.066$ , and inclination  $i = 0.11^\circ$ . With such an Earth-like orbit, one would expect this NEA to offer many efficient round-trip trajectory opportunities when in proper phase with respect to Earth.

Indeed, the direct approach in Reference 4 identified over 700,000 trajectory solutions that met the given constraints on Earth departure date, total round-trip flight time, and mission mass budget. In Reference 4 the non-dimensional parameter called the mass ratio, denoted by  $\alpha_m$ , was developed as a criterion for accessibility.  $\alpha_m$ , defined in Equation (15), is equal to the required initial total spacecraft mass (dry mass plus propellant) divided by the payload mass capability of a notional Ares V heavy-lift vehicle. The required propellant load for the spacecraft is a function of the required total mission  $\Delta V$  for a particular trajectory solution while the payload mass capability of the notional Ares V is a function of the Earth departure  $C_3$  for the particular trajectory solution. In Reference 4, an NEA had to offer at least one trajectory solution (within given constraints) for which  $\alpha_m \leq 1$ ; 2000 SG<sub>344</sub> offered over 700,000 such solutions, with Earth departure years between 2027 and 2030.

The new algorithm described herein was exercised on 2000 SG<sub>344</sub> and produced nearly 4,400 trajectory solutions that include a lunar flyby in the outbound trajectory segment departing Earth. However, these solutions occur on only four unique departure dates that fall between the years 2028 and 2030. It is important to note that only solutions which required  $\Delta V_{soi} \leq 0.01 \text{ km/s}$  were considered viable. This was to ensure that the outbound trajectory can be essentially smooth, with no major maneuver requirements. Table 1 compares the results of the new algorithm to the original results in Reference 4. Note that the restriction of including a lunar flyby dramatically reduced the number of viable trajectory solutions, reduced the window of viable of Earth departure dates from three years to two years, and increased the minimum available total round-trip flight time from three months to four months. However, also note that the minimum available mass ratio for the lunar flyby cases is about 2% lower than for the direct method. This provides our first demonstration that the inclusion of a lunar flyby in the outbound trajectory segment can indeed reduce the required propellant mass for the mission.

An example trajectory for 2000 SG<sub>344</sub> with initial condition given in Table 2 is shown in Figures 5(a) and 5(b) in a Sun-centered frame and in an Earth centered frame in Figures 5(c) and 5(d). Table 2 compares this specific trajectory solution generated by the new lunar flyby method to one generated by the direct method of Reference 4. Note that these trajectory solutions have nearly identical Earth departure dates (one day apart) and total round-trip flight times (151 versus 154 days). However, for the lunar flyby solution, the Earth departure  $C_3$  is considerably lower (and is in fact

**Table 1. Comparison of Lunar Flyby and Direct Trajectory Scan Results for NEA 2000 SG<sub>344</sub>**

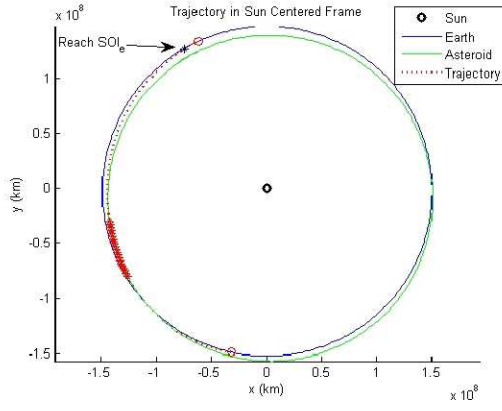
|                                  | <b>Lunar Flyby</b> | <b>Direct</b> |
|----------------------------------|--------------------|---------------|
| # of Viable Trajectory Solutions | 4,398              | 708,703       |
| Departure date                   | 2028 – 2030        | 2027 – 2030   |
| $tof_{min}$ (days)               | 128                | 96            |
| $tof_{max}$ (days)               | 360                | 360           |
| Min mass ratio $\alpha_m$        | 0.3429             | 0.3630        |
| Max mass ratio $\alpha_m$        | 0.9994             | 1.0000        |

commensurate with typical  $C_3$  requirements for lunar missions), the required  $\Delta V$  at Earth's Sphere of Influence is extremely small, and the total mission  $\Delta V$  requirement is significantly reduced. Thus the mass ratio for the lunar flyby case is lower since the available launch vehicle payload capacity is higher at the lower  $C_3$ , and the total required spacecraft propellant mass is less due to the reduced total mission  $\Delta V$  (all else being equal).

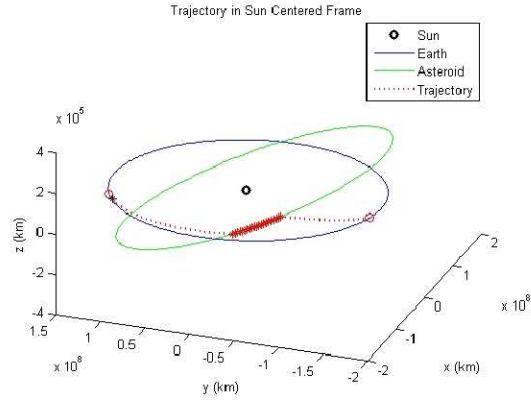
**Table 2. Comparison of Lunar Flyby and Direct Trajectory Solutions for NEA 2000 SG<sub>344</sub>**

|  | <b>flyby</b> | <b>direct</b> |
|--|--------------|---------------|
| $\theta_0$ ( $^\circ$ )                  | 0.00         | -             |
| $\alpha_{t0}$ ( $^\circ$ )               | 115.00       | -             |
| Launch date                              | 01/09/2029   | 01/10/2029    |
| tof (days)                               | 151          | 154           |
| $tof_{soi}$ (days)                       | 11           | -             |
| $tof_{arra}$ (days)                      | 68           | 76            |
| $tof_{stay}$ (days)                      | 20           | 20            |
| $tof_{arre}$ (days)                      | 52           | 58            |
| $C_3$ (km <sup>2</sup> /s <sup>2</sup> ) | -2.0390      | 1.6579        |
| $\Delta V_{soi}$ (km/s)                  | 0.0068       | -             |
| $\Delta V_{tot}$ (km/s)                  | 3.4312       | 4.5432        |
| mass ratio $\alpha_m$                    | 0.9331       | 0.9358        |

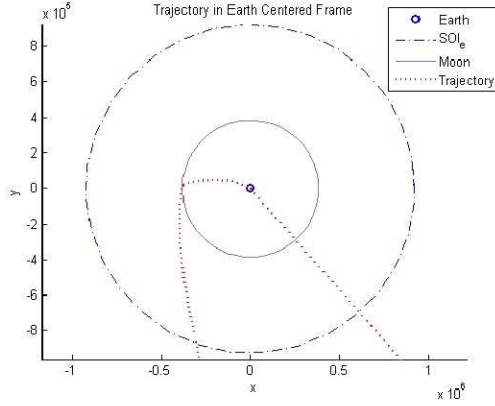
While these results are certainly encouraging, recall that thus far two major assumptions have been made. First, the trajectory of the Moon is modeled as being circular about the Earth, and in the ecliptic plane. Second, the lunar flyby is constructed with the freedom to place the Moon at precisely the correct location at the necessary time. However, subsequent analysis of the true lunar ephemeris revealed that Moon is never at the proper location on any of the four available departure dates. An example of this situation is shown in Figure 6 and clearly demonstrates that the Moon is rather far from the required location. Therefore, a round-trip trajectory utilizing a lunar flyby for Asteroid 2000SG344 is not possible in actuality. However, the results obtained with the fictitious



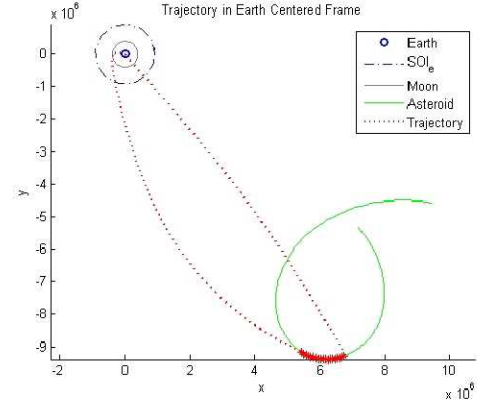
(a) Sun-Centered, Ecliptic Plane



(b) Sun-Centered, Inclined. Axis are Not at Same Scale



(c) Earth-Centered, Lunar Flyby Close-Up



(d) Earth-Centered, Round-Trip Trajectory

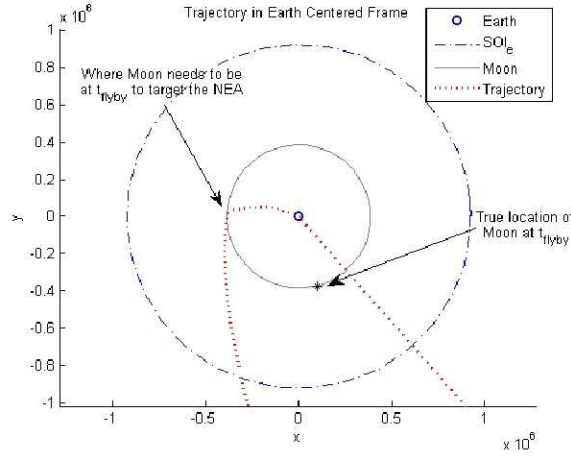
**Figure 5. Example Theoretical Trajectory for NEA 2000 SG<sub>344</sub>.**

lunar ephemeris served as a strong proof of concept and motivated the continuation of this research using the true lunar ephemeris.

## MOON'S EPHEMERIS IMPLEMENTATION

The true lunar ephemeris was incorporated into our lunar flyby algorithm following the successful proof of concept for NEA 2000 SG<sub>344</sub> using the fictitious lunar ephemeris. The previous methodology utilizing the fictitious model for the Moon's orbit still serves to provide good initial guesses for outbound trajectories that be may be able to utilize a lunar flyby en-route to a NEA. Thus the first step in the upgraded algorithm still involves constructing the trajectory sequence using the fictitious Moon and then checking the Moon's actual ephemeris to see whether its true position is within some tolerance of where the theoretical model indicates it needs to be on the particular flyby date. If so, then the initial guess can be used, in combination with the true lunar ephemeris, to compute the true trajectory solutions.

It is worth noting that the Moon's true state,  $\mathbf{x}$ , is in  $\mathbb{R}^3$ , which means that a lunar flyby can provide some inclination change for the spacecraft; modeling this was not possible with the planar model for the Moon's orbit. Also, the actual orientation of the Moon's orbit plane with respect to



**Figure 6. Moon's location is not coincident with trajectory.**

the ecliptic (a difference of about  $5^\circ$ ) is now accounted for when transforming coordinates from the Earth-centered to the Sun-centered frame.

### The Case of NEA 2003 LN<sub>6</sub>

Since NEA 2000 SG<sub>344</sub> was previously determined to have no lunar flyby opportunities when the true lunar ephemeris is accounted for (and since it was already accessible in the previous study without a lunar flyby), we selected a different candidate target from the list of marginally inaccessible NEAs presented in Reference 4. The goal was to show that a lunar flyby alone could make the NEA accessible according to the constraints and criteria of the previous study. We selected NEA 2003 LN<sub>6</sub> in the hopes that the true lunar position would be favorable for lunar flyby trajectory solutions. This NEA has an estimated size of 35 to 77 m, its orbit has a semi-major axis of 0.857 AU, its orbital eccentricity is 0.211, and its orbital inclination is  $0.633^\circ$ . The best-case (minimum) mass ratio trajectory solution found in the previous study for this NEA had an associated mass ratio value of 1.00325. Thus the required spacecraft propellant mass was just beyond the payload capacity of the assumed launch vehicle.

The first step was to employ the algorithm as described previously, using the fictitious model for the Moon's orbit. This produced 36 possible round-trip trajectory solutions which included a lunar flyby. These possible trajectory solutions were spread over four departure dates within the year 2025. Again, note that only solutions with  $\Delta V_{soi} \leq 0.1$  km/s were considered. Table 3 compares the results from the initial lunar flyby trajectory scan to the results of the direct method trajectory scan given for the previous study in Reference 4. Note that in the previous study only one minimum mass ratio trajectory solution was tabulated for each NEA found to be inaccessible. Note also that even using the fictitious model for the Moon's orbit, we already reduce the spacecraft propellant mass sufficiently to obtain an  $\alpha_m \leq 1$  solution for this NEA.

Out of the 36 possible trajectory solutions, 8 were found to have the Moon actually near the required location. One of these trajectory solutions was selected and the true lunar ephemeris was then used to compute a final lunar flyby trajectory solution for NEA 2003 LN<sub>6</sub>. The details of this trajectory solution are presented in Table 4. Note that the restriction of  $C_3$  in Eq. (8) having to be that of a Hohmann transfer is absent in the model with the Moon's true ephemerides, since



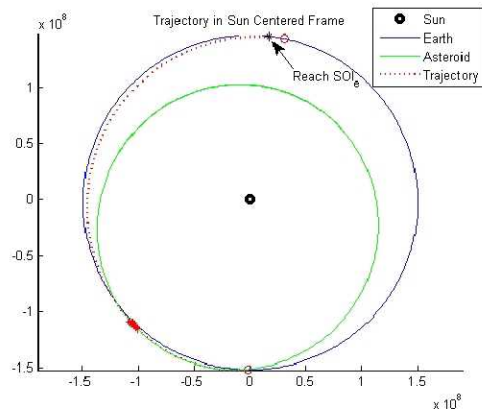
**Table 3. Comparison of Lunar Flyby and Direct Trajectory Scan Results for NEA 2003 LN<sub>6</sub>**

|                                  | <b>Lunar Flyby</b> | <b>Direct</b> |
|----------------------------------|--------------------|---------------|
| # of Viable Trajectory Solutions | 36                 | 0             |
| Departure date                   | 2025               | 2025          |
| $TOF_{min}$ (days)               | 188                | 204           |
| $TOF_{max}$ (days)               | 196                | -             |
| Min mass ratio                   | 0.9929             | 1.00325       |

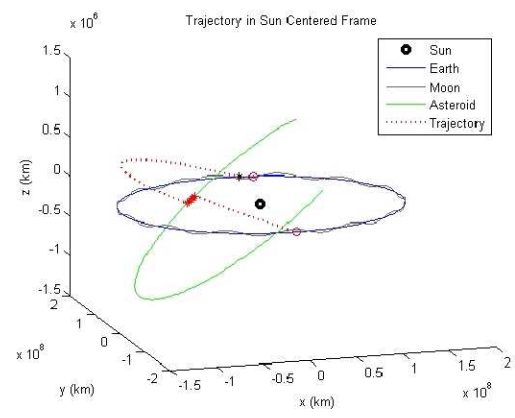
$C_3 = -2.0252 \text{ km}^2/\text{s}^2$  as shown in Table 4. The associated trajectory plots are presented in Figure 7. The comparison between the theoretical and true trajectories is shown in Figure 7(e).

**Table 4. Lunar Flyby Solution for NEA 2003 LN<sub>6</sub> Utilizing True Ephemerides Throughout**

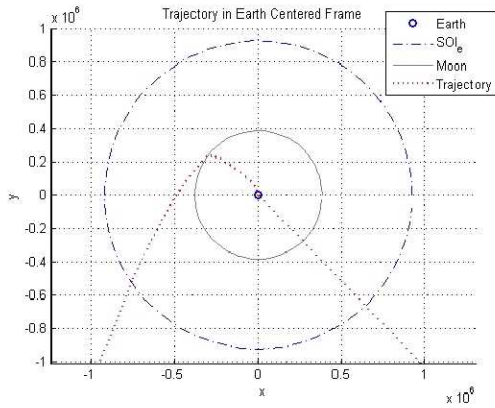
| <b>Quantity</b>                        | <b>Value</b> |
|--|--------------|
| Est. Size (m)                          | 35 - 77      |
| Launch date                            | 12/09/2025   |
| $\theta_0$                             | 331.61°      |
| $C_3 \text{ (km}^2/\text{s}^2\text{)}$ | -2.0252      |
| $TOF_{tot}$ (days)                     | 198.4        |
| $tof_{soi}$ (days)                     | 11.91        |
| $tof_{dep}$ (days)                     | 134.49       |
| $tof_{stay}$ (days)                    | 4            |
| $tof_{ret}$ (days)                     | 48           |
| $\Delta V_{soi}$ (km/s)                | 0.1122       |
| $\Delta V_{tot}$ (km/s)                | 3.6428       |
| $\alpha_m$                             | 0.9997       |



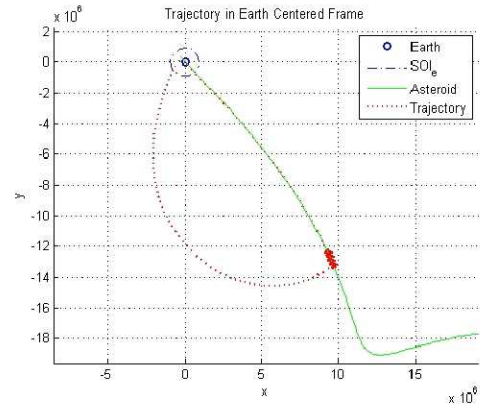
(a) Sun Centered, Ecliptic Plane



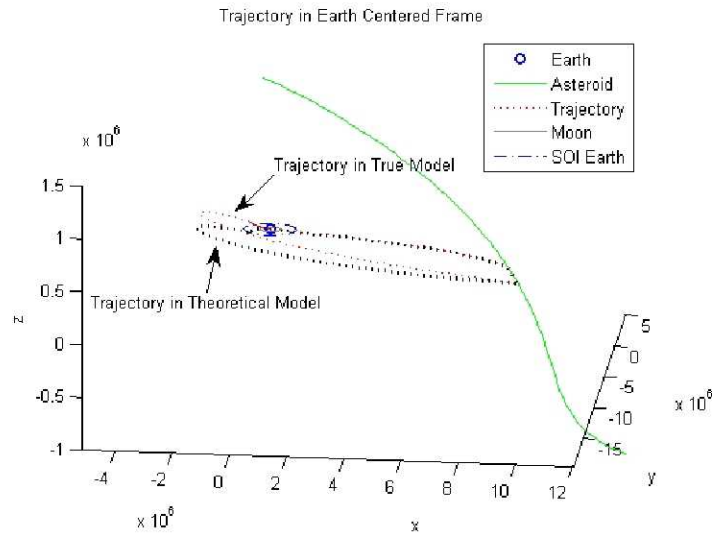
(b) Sun Centered, Inclined. Axis are Not at Same Scale



(c) Earth Centered, Flyby



(d) Earth Centered



(e) Comparison of Trajectory Solutions Using Theoretical and True Lunar Ephemerides

**Figure 7. Plots of Round-trip Trajectory to 2003 LN<sub>6</sub> Using True Lunar Ephemeris**

## CONCLUSION

In this paper we present a new general methodology for designing round-trip trajectories to NEAs in which lunar gravity assist is utilized to aid the outbound trajectory segment. Our results were compared to those from a previous study<sup>4</sup> in which round-trip trajectories were designed without the lunar gravity assist. The objective was to gain sufficient velocity from Moon's gravity to reduce the total spacecraft propellant mass required for a round-trip mission to an NEA. Asteroid 2003 LN<sub>6</sub>, an NEA categorized as marginally inaccessible in Reference 4, was made accessible (according to the previous study criteria) via a lunar flyby using the new algorithm presented herein. The lunar flyby method accomplished this by reducing the post-Earth departure  $\Delta V$  for the mission by only  $\approx 2\%$ , while also reducing the Earth departure  $C_3$  to be commensurate with what is typically required for a lunar mission.

The new algorithm minimizes the magnitude of the post-lunar flyby maneuver required to reach the NEA,  $\Delta V_{soi}$ . In future work we will seek to eliminate the need for that maneuver, i.e., achieve  $\Delta V_{soi} = 0$ . We also plan to optimize the trajectories such that both  $\Delta V_{tot}$  and  $tof_{tot}$  are minimized, while also converting all the impulsively modeled maneuvers to finite burns. The possibility of a lunar flyby in the return segment from an NEA of the trajectory will be studied as well to determine if other NEAs can be made accessible that are now classified as inaccessible.

## ACKNOWLEDGMENT

The authors wish to thank Emergent Space Technologies, Inc. for funding this research.

## REFERENCES

- [1] Committee of Near-Earth-Object and National Research Council, *Defending Planet Earth: Near-Earth-Object Surveys and Hazard Mitigation Strategies*. Washington, D.C.: National Academic Press, 2010.
- [2] S. Wagner and B. Wie, "Target Asteroid Selection for Human Exploration of Near Earth Objects," AAS 10-249, 2010.
- [3] D. Zimmerman, S. Wagner, and B. Wie, "The First Human Asteroid Mission: Target Selection and Conceptual Mission Design," *Proc. AIAA/AAS Astrodynamics Specialist Conference*, Toronto, Canada, August 2010.
- [4] B. W. Barbee and *et al.*, "A Comprehensive Ongoing Survey of the Near-Earth Asteroid Population for Human Mission Accessibility," *Proc. AIAA/AAS Astrodynamics Specialist Conference*, Toronto, Canada, August 2010.
- [5] M. H. Kaplan, *Modern Spacecraft Dynamics & Control*. John Wiley & Sons, Inc., 1976.
- [6] R. R. Bate, D. D. Mueller, and J. E. White, *Fundamentals of Astrodynamics*. Dover Publications, Inc., 1971.
- [7] J. P. Sumrall, "Ares V Overview, <http://event.arc.nasa.gov/aresv-sss/home/ppt/AresV-sss/SAT/am/4Sumrall/7567AresVSolarSysWorkshop.pdf>," *Ares V Solar System Science Workshop*, April 2008.